

Shear Design

Based on

Sectional Model / Modified
Compression Field Theory

5.8.2 General Requirements

$$V_u > 0.5\phi(V_c + V_p) \quad (5.8.2.4-1)$$

where:

V_u = factored shear force, kips

V_c = nominal shear capacity of the concrete, kips

V_p = vertical component of the prestressing force, kips

ϕ = resistance factor (LRFD 5.5.4.2)

5.8.3.3 Nominal Shear Resistance

$$V_n = V_c + V_s + V_p \quad (5.8.3.3-1)$$

$$V_n = 0.25 f'_c b_v d_v + V_p \quad (5.8.3.3-2)$$

in which:

$$V_c = 0.0316 \beta \sqrt{f'_c} b_v d_v \quad (5.8.3.3-3)$$

$$V_s = \frac{A_v f_y d_v (\cot \theta + \cot \alpha) \sin \alpha}{s} \quad (5.8.3.3-4)$$

where:

b_v = effective web width taken as the minimum web width within the depth d_v as determined in Article 5.8.2.9 (in.)

d_v = effective shear depth as determined in Article 5.8.2.9 (in.)

s = spacing of stirrups (in.)

β = factor indicating ability of diagonally cracked concrete to transmit tension as specified in Article 5.8.3.4

θ = angle of inclination of diagonal compressive stresses as determined in Article 5.8.3.4 ($^\circ$)

α = angle of inclination of transverse reinforcement to longitudinal axis ($^\circ$)

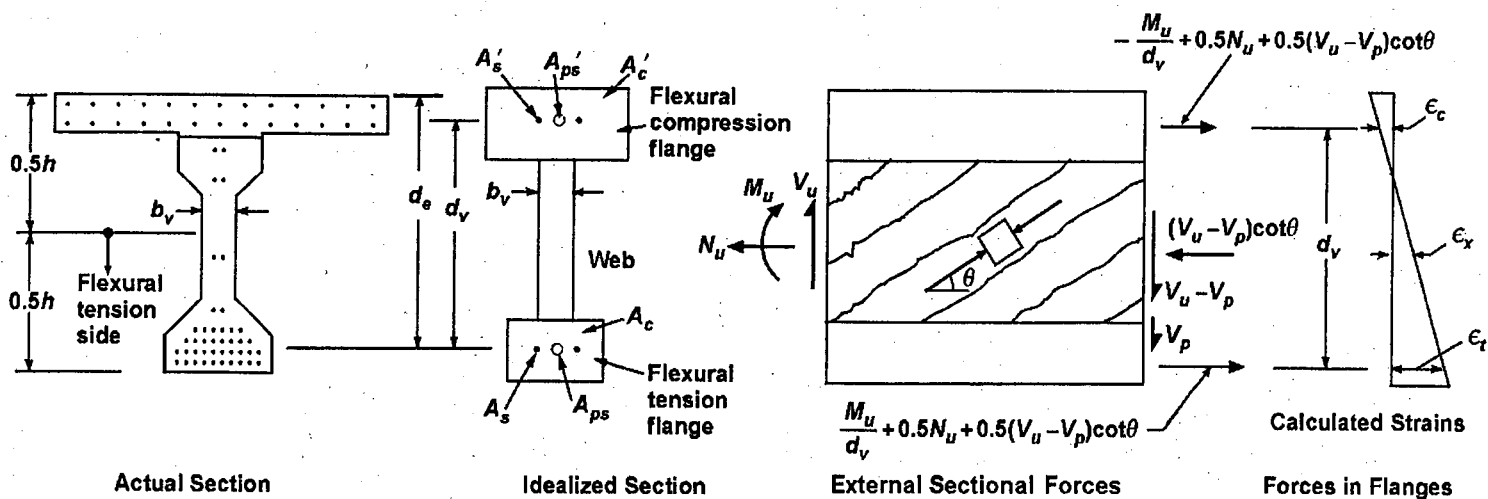
A_v = area of shear reinforcement within a distance s (in.²)

V_p = component in the direction of the applied shear of the effective prestressing force; positive if resisting the applied shear (kip)

5.8.3.4 Determination of β and θ

Table 5.8.3.4.2-1 Values of θ and β for Sections with Transverse Reinforcement.

$\frac{v_u}{f'_c}$	$\epsilon_x \times 1,000$								
	≤ -0.20	≤ -0.10	≤ -0.05	≤ 0	≤ 0.125	≤ 0.25	≤ 0.50	≤ 0.75	≤ 1.00
≤ 0.075	22.3 6.32	20.4 4.75	21.0 4.10	21.8 3.75	24.3 3.24	26.6 2.94	30.5 2.59	33.7 2.38	36.4 2.23
≤ 0.100	18.1 3.79	20.4 3.38	21.4 3.24	22.5 3.14	24.9 2.91	27.1 2.75	30.8 2.50	34.0 2.32	36.7 2.18
≤ 0.125	19.9 3.18	21.9 2.99	22.8 2.94	23.7 2.87	25.9 2.74	27.9 2.62	31.4 2.42	34.4 2.26	37.0 2.13
≤ 0.150	21.6 2.88	23.3 2.79	24.2 2.78	25.0 2.72	26.9 2.60	28.8 2.52	32.1 2.36	34.9 2.21	37.3 2.08
≤ 0.175	23.2 2.73	24.7 2.66	25.5 2.65	26.2 2.60	28.0 2.52	29.7 2.44	32.7 2.28	35.2 2.14	36.8 1.96
≤ 0.200	24.7 2.63	26.1 2.59	26.7 2.52	27.4 2.51	29.0 2.43	30.6 2.37	32.8 2.14	34.5 1.94	36.1 1.79
≤ 0.225	26.1 2.53	27.3 2.45	27.9 2.42	28.5 2.40	30.0 2.34	30.8 2.14	32.3 1.86	34.0 1.73	35.7 1.64
≤ 0.250	27.5 2.39	28.6 2.39	29.1 2.33	29.7 2.33	30.6 2.12	31.3 1.93	32.8 1.70	34.3 1.58	35.8 1.50



5.8.2.9 Shear Stress on Concrete

The shear stress on the concrete shall be determined as:

$$v_u = \frac{|V_u - \phi V_p|}{\phi b_v d_v} \quad (5.8.2.9-1)$$

where:

b_v = effective web width taken as the minimum web width, measured parallel to the neutral axis, between the resultants of the tensile and compressive forces due to flexure, or for circular sections, the diameter of the section, modified for the presence of ducts where applicable (in.)

d_v = effective shear depth taken as the distance, measured perpendicular to the neutral axis, between the resultants of the tensile and compressive forces due to flexure; it need not be taken to be less than the greater of $0.9 d_e$ or $0.72h$ (in.)

ϕ = resistance factor for shear specified in Article 5.5.4.2

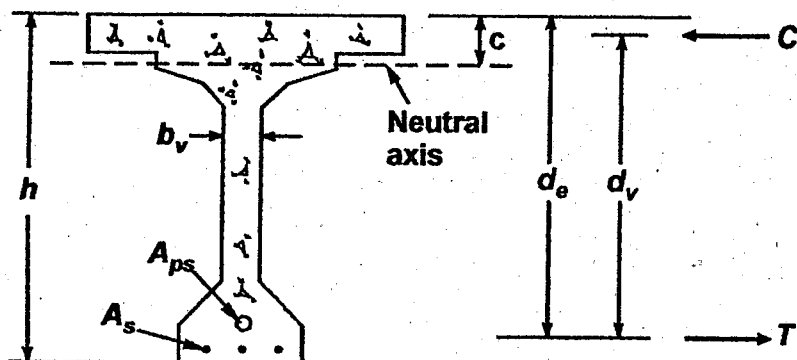


Figure C5.8.2.9-1 Illustration of the Terms b_v and d_v .

5.8.3.4.2 Calculate Strain ϵ_x

- If the section contains at least the minimum transverse reinforcement as specified in Article 5.8.2.5:

$$\epsilon_x = \frac{\left(\frac{|M_u|}{d_v} + 0.5N_u + 0.5|V_u - V_p| \cot \theta - A_{ps}f_{po} \right)}{2(E_s A_s + E_p A_{ps})} \quad (5.8.3.4.2-1)$$

The initial value of ϵ_x should not be taken greater than 0.001.

- If the section contains less than the minimum transverse reinforcement as specified in Article 5.8.2.5:

$$\epsilon_x = \frac{\left(\frac{|M_u|}{d_v} + 0.5N_u + 0.5|V_u - V_p| \cot \theta - A_{ps}f_{po} \right)}{E_s A_s + E_p A_{ps}} \quad (5.8.3.4.2-2)$$

The initial value of ϵ_x should not be taken greater than 0.002.

- If the value of ϵ_x from Eqs. 1 or 2 is negative, the strain shall be taken as:

$$\epsilon_x = \frac{\left(\frac{|M_u|}{d_v} + 0.5N_u + 0.5|V_u - V_p| \cot \theta - A_{ps}f_{po} \right)}{2(E_c A_c + E_s A_s + E_p A_{ps})} \quad (5.8.3.4.2-3)$$

where:

A_c = area of concrete on the flexural tension side of the member as shown in Figure 1 (in.²)

A_{ps} = area of prestressing steel on the flexural tension side of the member, as shown in Figure 1 (in.²)

A_s = area of nonprestressed steel on the flexural tension side of the member at the section under consideration, as shown in Figure 1. In calculating A_s for use in this equation, bars which are terminated at a distance less than their development length from the section under consideration shall be ignored (in.²)

f_{po} = a parameter taken as modulus of elasticity of prestressing tendons multiplied by the locked-in difference in strain between the prestressing tendons and the surrounding concrete (ksi). For the usual levels of prestressing, a value of 0.7 f_{pu} will be appropriate for both pretensioned and post-tensioned members

N_u = factored axial force, taken as positive if tensile and negative if compressive (kip)

M_u = factored moment, not to be taken less than $V_u d_v$ (kip-in.)

V_u = factored shear force (kip)

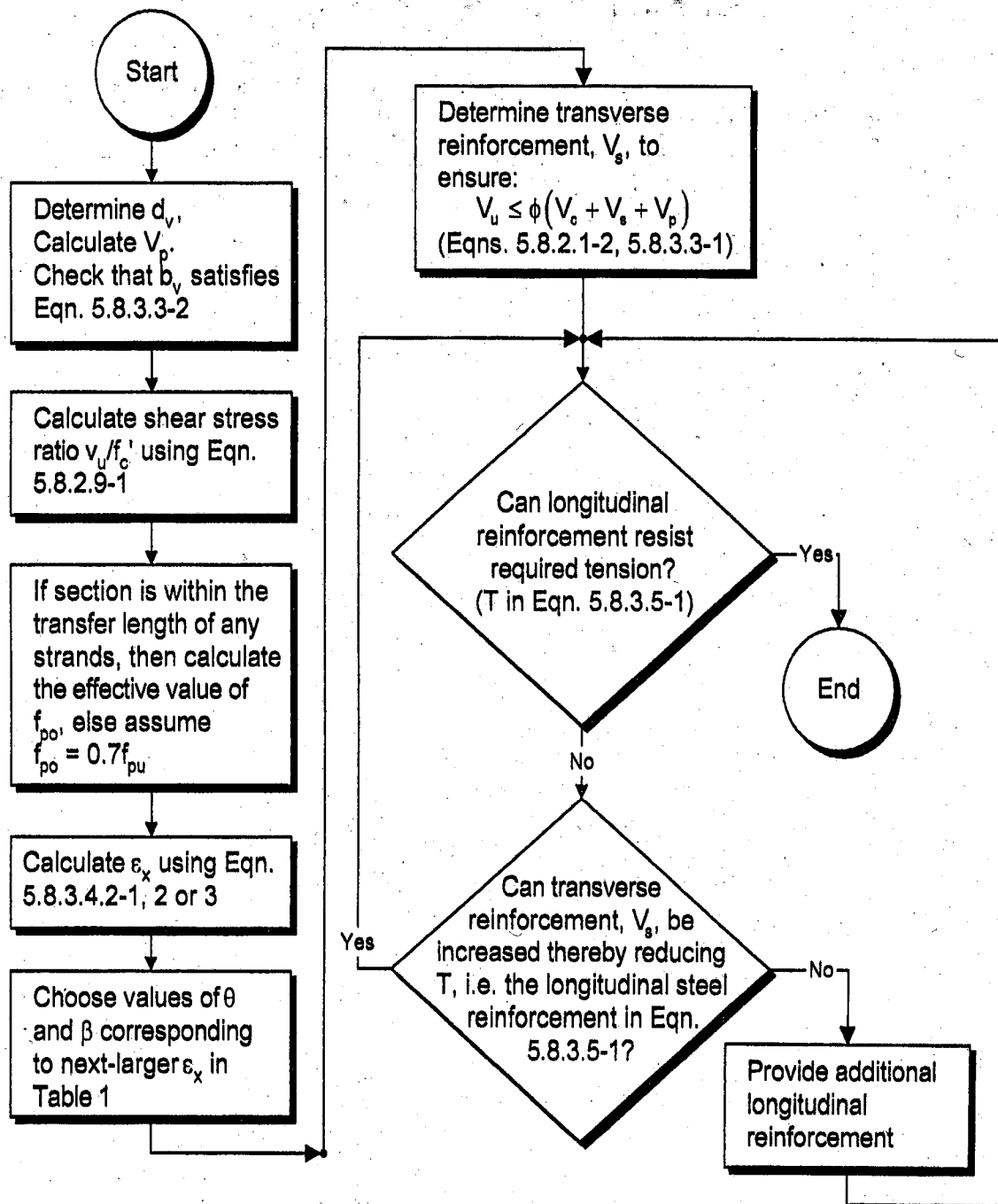


Figure C5.8.3.4.2-5 Flow Chart for Shear Design of Section Containing at Least Minimum Transverse Reinforcement.

5.8.3.5 Longitudinal Reinforcement

At each section the tensile capacity of the longitudinal reinforcement on the flexural tension side of the member shall be proportioned to satisfy:

$$A_{ps}f_{ps} + A_s f_y \geq \frac{|M_u|}{d_v \phi} + 0.5 \frac{N_u}{\phi} + \left(\left| \frac{V_u}{\phi} - V_p \right| - 0.5V_s \right) \cot \theta$$

(5.8.3.5-1)

where:

V_s = shear resistance provided by the transverse reinforcement at the section under investigation as given by Eq. 5.8.3.3-4, except V_s shall not be taken as greater than V_u/ϕ (kip)

θ = angle of inclination of diagonal compressive stresses used in determining the nominal shear resistance of the section under investigation as determined by Article 5.8.3.4 (°)

$\phi_f \phi_v \phi_c$ = resistance factors taken from Article 5.5.4.2 as appropriate for moment, shear and axial resistance

At the inside edge of the bearing area of simple end supports to the section of critical shear, the longitudinal reinforcement on the flexural tension side of the member shall satisfy:

$$A_s f_y + A_{ps} f_{ps} \geq \left(\frac{V_u}{\phi_v} - 0.5V_s - V_p \right) \cot \theta \quad (5.8.3.5-2)$$

Eqs. 1 and 2 shall be taken to apply to sections not subjected to torsion. Any lack of full development shall be accounted for.

5.8.2.5 Minimum Transverse Reinforcement

Except for segmental post-tensioned concrete box girder bridges, where transverse reinforcement is required, as specified in Article 5.8.2.4, the area of steel shall satisfy:

$$A_v \geq 0.0316 \sqrt{f'_c} \frac{b_v s}{f_y} \quad (5.8.2.5-1)$$

where:

A_v = area of a transverse reinforcement within distance s (in.²)

b_v = width of web adjusted for the presence of ducts as specified in Article 5.8.2.9 (in.)

s = spacing of transverse reinforcement (in.)

f_y = yield strength of transverse reinforcement (ksi)

5.8.2.7 Maximum Spacing of Transverse Reinforcement

The spacing of the transverse reinforcement shall not exceed the maximum permitted spacing, s_{max} , determined as:

- If $v_u < 0.125 f'_c$, then:

$$s_{max} = 0.8d_v \leq 24.0 \text{ in.} \quad (5.8.2.7-1)$$

- If $v_u \geq 0.125 f'_c$, then:

$$s_{max} = 0.4d_v \leq 12.0 \text{ in.} \quad (5.8.2.7-2)$$

where:

v_u = the shear stress calculated in accordance with 5.8.2.9 (ksi)

d_v = effective shear depth as defined in Article 5.8.2.9 (in.)

5.8.4 Interface Shear Transfer – Horizontal Shear

The nominal shear resistance of the interface plane shall be taken as:

$$V_n = cA_{cv} + \mu[A_{vf}f_y + P_c] \quad (5.8.4.1-1)$$

The nominal shear resistance, V_n , used in the design shall not be greater than the lesser of:

$$V_n \leq 0.2f'_cA_{cv}, \text{ or} \quad (5.8.4.1-2)$$

$$V_n \leq 0.8A_{cv} \quad (5.8.4.1-3)$$

where:

V_n = nominal shear resistance (kip)

A_{cv} = area of concrete engaged in shear transfer (in.²)

A_{vf} = area of shear reinforcement crossing the shear plane (in.²)

f_y = yield strength of reinforcement (ksi)

c = cohesion factor specified in Article 5.8.4.2 (ksi)

μ = friction factor specified in Article 5.8.4.2

P_c = permanent net compressive force normal to the shear plane; if force is tensile, $P_c = 0.0$ (kip)

f'_c = specified 28-day compressive strength of the weaker concrete (ksi)

Reinforcement for interface shear between concretes of slab and beams or girders may consist of single bars, multiple leg stirrups, or the vertical legs of welded wire fabric. The cross-sectional area, A_{vf} , of the reinforcement per unit length of the beam or girder should satisfy either that required by Eq. 1 or:

$$A_{vf} \geq \frac{0.05 b_v}{f_y} \quad (5.8.4.1-4)$$

where:

b_v = width of the interface (in.)

The minimum reinforcement requirement of A_{vf} may be waived if V_n/A_{cv} is less than 0.100 ksi.